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LOGICAL PROPERTIES OF IMAGINATION *

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Abstract

Inspired by Niiniluoto's account of the logic of imagination, this work proposes a combined logic able to deal with interactions of imagination, conception and possibility. It combines Descartes' view according to which imagination implies conception with Hume's view according to which both imagination and conception imply possibility.

1 Introduction

This study argues that imagination and conception are two weak kinds of possibility, although they are intrinsically connected. For this purpose, we have constructed a logic in which the relations between them is clearly defined. This system characterizes the minimal logic of imagination and related notions.

In order to understand the relations between imagination, conception and possibility, it is important to note that R.Descartes in [2] already proposed a distinction between them: for him, imagination implies conception, while conception does not imply imagination.[1] In this sense, we have two distinct levels of mental acts: imagining and conceiving. D.Hume in [6] defends an empiricist notion of

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[1] We can recognize this same distinction in N. Vasiliev (see [11]): he states that we can conceive an *n*-dimensional logic but we cannot imagine it.

imagination, which we accept without restriction. Gendler and Hawthorne in [4], as well as R. Sorensen in [10], recognized in Hume a reduction of conceiving to imagining.

We accept that imagination and conception are distinct concepts in the sense defended by R. Descartes. But we also accept that D. Hume is correct while announcing that both imagination and conceiving imply possibility. Indeed, we have proposed a sound and complete combined logic showing that both Descartes' view and Hume's view are compatible.

Philosophical studies on imagination, conception and possibility (as well as their interactions) are frequent in the history of philosophy: from Descartes, Hume and Vasiliev to very recent studies as those which can be found in the book edited by Gendler and Hawthorne (see [4]) and also in the book edited by Nichols (see [8]), where many contemporary philosophers study the subject (Chalmers, Yablo, Fine, Stalnaker, Sorensen etc). However, none of the mentioned philosophers has proposed a logic of imagination and conception. Indeed, R. Sorensen vaguely proposed a "logic" of meta-conception. He even considered a conceivability operator and formalized it as C. Moreover, the author studied an interaction of conceivability and possibility, but he has reduced conceivability to conception.

Any attempt to formalize the concepts of imagination and conception should take into consideration the first and unique proposal to elaborate a logic of imagination developed by I. Niiniluoto in [9]. His approach has many merits, but also some gaps. His idea consists in exploring imagination as a modal operator in the same sense that J. Hintikka in [5] studied the notions of knowledge and belief. There are many kinds of epistemic notions which are usually called *propositional attitudes*. The first philosopher who developed a logical and formal account of epistemic notions is J. Hintikka in [5]. His work on epistemic logic has been important for all later work on the subject.

Considering imagination as an operator, Niiniluoto was able to investigate properties of it. He introduces the imagination operator I in order to formalize sentences of the form *an agent imagines that* φ. Therefore, he proposes the following set of axioms in Hilbert-style presentation plus one inference rule (where $\vdash$ means the standard notion of syntactical logical consequence - the basis of the system is classical logic):

1. $I(\varphi \rightarrow \psi) \rightarrow (I\varphi \rightarrow I\psi)$;

2. $I(\varphi \wedge \psi) \leftrightarrow (I\varphi \wedge I\psi)$;

3. From $\vdash \varphi$, we derive $\vdash I\varphi$;

Niiniluoto states that the above axioms are consequences of the following semantical condition: an agent a imagines φ in w if and only if φ is true in all possible worlds compatible with what a imagines in w. The author also argues that $I\varphi \rightarrow \varphi$ does not hold while $I\varphi \rightarrow \Diamond\varphi$ holds. So, imagination is viewed semantically as a kind of $\Box$-operator. Niiniluoto's approach to logical aspects of imagination is insufficient, considering that:

1. It examines a case of modal interaction without appealing to combining logics;

2. Metalogical properties of the logic are not examined;

3. It does not distinguish between imagination and conception;

4. Not intuitive inference rule.

Thus, given the insufficiency of Niiniluoto's approach, one has to search for a plausible logic of imagination. In the same way Niiniluoto has introduced a new operator to reason about imagination, we can go on and introduce another operator to reason about conception. So, we introduce in the object language an operator C formalizing sentences of the type *an agent* a *conceives* φ as $C_a\varphi$. Thus, we are able to construct an adequate environment to discuss about the distinctions and similarities between imagination and conception.

2 Imagination and conception as diamonds

Imagination is a faculty of minds able to generate images of objects (be they real or not). Whenever an agent imagines something (in this case the *something* is a particular proposition), we say that there is an *act of imagining*. We take imagination in an empirical fashion, following Hume's approach. This means that acts of imagination are connected to previous sense data. For theoretical reasons, we assume that the content of a given act of imagination is a proposition. Then, we speak about *propositional imagination* in the sense of [8].

Conception (or pure intellection, in Descartes' terminology) is a faculty of minds able to generate understanding of concepts and/or propositions. It is not necessarily related to images, but to comprehension. Whenever an agent conceives something (a proposition), we say that there is an *act of conceiving*.

Evidently, both imagination and conception are ways of representing things (representation mechanisms, acts of thought), and any act of imagining is an act of conceiving, but conception cannot be reduced to imagination. Our favorite example to elucidate this topic is the Cartesian one. Descartes in [2] has argued that although the mind cannot imagine a geometrical structure with a thousand sides, it can conceive it. In this sense, conception is a kind of understanding, a notion much more general than imagination.

We assume that agents imagine propositions, but they can imagine more (or less). Take these examples:

1. John imagines that it is raining in Manhattan;

2. John imagines Manhattan;

In (1) the content of imagination is the proposition *it is raining in Manhattan*, while in (2) the content of imagination is only *Manhattan*. Both can be understood as propositional imagination because although the first one is a proposition and the second a mere object, *Manhattan* can be viewed as a collection of properties and can be defined, therefore, as a collection of propositions. So, each object corresponds in some sense to a given proposition. Given any object, for instance, *Manhattan* we can associate to it a proposition: *There exists Manhattan.*[2] This lead us to the view according to which all kinds of imagination can be reduced to propositional imagination.

Generally speaking, imagination is a weaker concept than that of conception which, in its turn, is a weaker concept than that of logical possibility. Comparisons between imagination, conception and possibility can find a good environment in modal logics.

There are many notions and philosophical distinctions concerning the concept of possibility. Basically, there are two kinds of possibility: empirical and logical. Empirical possibility depends of a given context: given a context X (a scientific area for instance), one can define the X-possibility. In this sense, some authors say there are things physically-possible, biologically-possible and so on. All these kinds of possibility can be reduced to what we call here empirical possibility. In this sense, a proposition is empirically-possible if and only if it does not contradict the underlying empirical theory. Logical possibility is something more general and it is our favorite notion of possibility. But there are indeed many ways one could define logical possibility. Consider a standard interpretation of logical possibility using the symbol $\Diamond$. Thus we can formalize sentences of the form "φ is possible" by $\Diamond\varphi$. Take also a Kripke frame. The notion of imagination is studied considering its relation with the notion of logical possibility. In order to define imagination one needs to use a notion of possibility able to capture in some sense the content of imaginative acts. In this sense we have (where R is an accessibility relation without restrictions):

$$(\text{MODAL}) w \vDash \Diamond\varphi \text{ if and only if } \exists w' \text{ such that } wRw', w' \vDash \varphi$$

This notion has been made clear by the developments of modal logic. It contains the key idea of possible worlds and given that possible worlds are important

[2] We can add quantifiers to the logic of imagination in order to prove this fact. Thanks to Niiniluoto for this remark.

for a part of the constitution of what is imagination, it follows that it is the choice in a logical theory trying to model the concept of imagination. The modal criterion of possibility can be applied to model imagination and conception. At the same time, these can be used to determine whether something is logically possible or not, playing a role of guides to possibility (See discussions on whether conceivability/imaginability are guides to possibility in [4]).

3 The logics of imagination, conception and possibility

The first thing to be said concerning a logic of imagination is that imagination can be treated as a modal notion. In this sense, it has some connections with different kinds of modality. It has a diamond-like truth condition. A logic of imagination is constructed using I (imagination), C (conception) and $\Diamond$ (possibility). Combined they can give rise to interesting philosophical interactions: $\Diamond I$ (imaginability) and $\Diamond C$ (conceivability), for instance.

Many formulas containing interactions of these notions can be presented. Studying how these operators behave is one of the motivations for a logic of imagination and related notions. While it is very difficult to compare the concept of imagination with necessity, it is very easy to compare it with possibility, given that imagination implies possibility seems to be plausible, but possibility does not imply imagination. Moreover, imagination does not imply necessity and vice-versa.

As we said, here we have to introduce in the language of modal logic for possibility a new modal operator in the same style of Niiniluoto. We represent this new operator by I and call it the *imagination operator*. We want our logic of imagination to respect some basic and most important properties of imagination, and we also want that it denies strange properties as for instance the property according to which imagination implies truth and that possibility implies imagination. So we have to build a formal system taking all these facts into consideration.

Consider the language L of classical propositional logic (**CPL**) defined by the structure $L =< \wedge, \vee \rightarrow, \neg >$. Adding to this language the imagination operator, we generate the minimal language to describe imagination, let's call it $L_I =< \wedge, \vee \rightarrow, \neg, I >$. We repeat the procedure, taking now C for conception operator and $\Diamond$ for possibility in order to get languages L_C and $L_\Diamond$, respectively. So, we have three languages: one for imagination, other for conception and another for possibility.

For each language, consider $\sharp \in \{I, C, \Diamond\}$. Then, we define three axiomatic systems using a diamond-based presentation of **K** as the one proposed by Blackburn, De Rijke and Venema in [1] in order to guarantee normality:

1. $\sharp\bot \leftrightarrow \bot$;
2. $\sharp(\varphi \vee \psi) \leftrightarrow (\sharp\varphi \vee \sharp\psi)$;
3. $\vdash \varphi \rightarrow \psi$ then $\vdash \sharp\varphi \rightarrow \sharp\psi$.

Replacing uniformly each occurrence of $\sharp$ by I, C or $\Diamond$, we have three axiomatic systems. Therefore, from the syntactical viewpoint, we do not have any criteria to distinguish between imagination, conception and possibility. For each operator, we can build the related dual. In this sense, we have the dual of imagination $\Box_I\varphi$ (This dual is exactly Niiniluoto's imagination operator). This operator satisfies all standard axioms for $\Box$ and it is very useful in the completeness proof. The same holds for duals of conception and possibility.

For each axiomatic system, we have a respective frame such that for each $\sharp \in \{I, C, \Diamond\}$ we define $F_\sharp =< W, R_\sharp >$. Thus, imagination, conception and possibility have the following truth-condition:

$$w \Vdash \sharp\varphi \text{ if and only if } \exists w' \text{ such that } wR_\sharp w', w' \Vdash \varphi.$$

In the same way, semantically, we do not have elements to distinguish between imagination, conception and possibility. For each instantiantion of $\sharp$, we have a sound and complete logic with respect to its class of all frames. Let's call these logics K_I, K_C and $K_\Diamond$. So, up to now, there are no tools to interact and reason about each operator in connection with another. However, the situation can change extending our logics by fusions and adding interaction axioms.

In order to define interactions of imagination, conception and possibility, let's take the fusion of the languages, axiomatic systems and frames. In this sense we have a logic

$$K_I \oplus K_C \oplus K_\Diamond$$

which is sound and complete with respect to the class of frames of the form

$$F = < W, R_I, R_C, R_\Diamond >$$

The proof of this could be constructed by canonical models or by preservation of completeness by fusions as developed by Fine and Schurz in [3] and Wolter and Kracht in [7]. However, even in the fusion, we cannot distinguish between imagination, conception and possibility.

We need to add interaction axioms in order to reason about the distinctions mentioned above. We use basic philosophical intuitions to determine which are the interesting axioms to be added in the fusion. In this sense, considering that imagination implies conception, we define Descartes-Vasiliev law:

$$I\varphi \rightarrow C\varphi$$

Considering that conception implies possibility, and imagination implies possibility, we define the so-called laws of Hume:

$$LH = \begin{cases} C\varphi \rightarrow \Diamond\varphi; \\ I\varphi \rightarrow \Diamond\varphi. \end{cases}$$

Obviously, the second law of Hume is a consequence of the law of Descartes-Vasiliev and the first law of Hume. Thus, we express the relations between these concepts, expanding the fusion in the following way:

$$K_I \oplus K_C \oplus K_\Diamond \oplus (I\varphi \to C\varphi) \oplus (C\varphi \to \Diamond\varphi)$$

Let's call this logic **IMAG**. This system is sound and complete with respect to the class of all frames F such that $R_I \subseteq R_C \subseteq R_\Diamond$. We denote this class of frames as $F^\subseteq$. **IMAG** has many very interesting properties. Before checking them, let's see that **IMAG** is sound and complete with respect to $F^\subseteq$.

For soundness, we need to verify that both Descartes-Vasiliev law and the first law of Hume - interaction axioms added to the fusion - are valid. That the inference rules and other parts of the logic preserve validity is evident. To check that the law of Descartes-Vasiliev is valid, take $w \vDash I\varphi$ but $w \nvDash C\varphi$. Thus:

1. $w \vDash I\varphi \Longleftrightarrow \exists w^{'}$ such that $wR_I w^{'}$, $w^{'} \vDash \varphi$;
2. $w \nvDash C\varphi \Longleftrightarrow \forall w^{'}$ such that $wR_C w^{'}$, $w^{'} \nvDash \varphi$;

Given that $R_I \subseteq R_C$, it follows the desired result. The same argument applies to the first law of Hume.

For completeness, we need to show that all **IMAG** valid formulae are theorems. We can proceed by canonical models method adapting standard proofs. While proving completeness we need to consider dual operators of I, C and $\Diamond$ as well their properties, which behave like $\Box$ operators.

The result presented here can be shown to be a special case of a general result on interaction axioms. Consider a hierarchy of diamond operators:

$$\Diamond_1, \Diamond_2, ..., \Diamond_n$$

such that each $\Diamond_i$ is weaker than a $\Diamond_j$ if $i \leq j$.

Each language containing a $\Diamond_i$ generates a logic which only modal operator is $\Diamond_i$, for some i. Thus, we define a fusion

$$K_{\Diamond_1} \oplus K_{\Diamond_2} \oplus ... \oplus K_{\Diamond_n}$$

This fusion can be expanded by the addition of finite many interaction axioms in the following way:

$$K_{\Diamond_1} \oplus K_{\Diamond_2} \oplus ... \oplus K_{\Diamond_n} \oplus (\Diamond_1\varphi \rightarrow \Diamond_2\varphi) \oplus ... \oplus (\Diamond_{n-1}\varphi \rightarrow \Diamond_n\varphi)$$

The above fusion is sound and complete with respect to the class of frames $F_{\Diamond}^{\subseteq} = < W, R_{\Diamond_1}, R_{\Diamond_2}, ..., R_{\Diamond_n} >$ such that $R_{\Diamond_1} \subseteq R_{\Diamond_2} \subseteq ... \subseteq R_{\Diamond_n}$.

4 Properties of IMAG

Now we answer questions posed in the literature on the relations between imagination, conception and possibility. We note that derived notions such as imaginability and conceivability cannot be reduced to imagination and conception, respectively, if some restrictions are not added to the accessibility relations. The following are valid in **IMAG**:

Interactions	Distributions	Connections
$I\varphi \rightarrow C\varphi$	$I(\varphi \wedge \psi) \rightarrow (I\varphi \wedge I\psi)$	$I\Diamond\varphi \leftrightarrow \Diamond I\varphi$
$C\varphi \rightarrow \Diamond\varphi$	$C(\varphi \wedge \psi) \rightarrow (C\varphi \wedge C\psi)$	$IC\varphi \leftrightarrow CI\varphi$
$I\varphi \rightarrow \Diamond\varphi$	$\Diamond(\varphi \wedge \psi) \rightarrow (\Diamond\varphi \wedge \Diamond\psi)$	$C\Diamond\varphi \leftrightarrow \Diamond C\varphi$

These would be valid if **IMAG**-frames were reflexive and transitive, respectively:

Reflexive	Transitive
$\varphi \rightarrow I\varphi$	$\Diamond C\varphi \rightarrow \Diamond\varphi$
$\varphi \rightarrow C\varphi$	$\Diamond I\varphi \rightarrow \Diamond\varphi$
$\varphi \rightarrow \Diamond\varphi$	$\Diamond\Diamond\varphi \rightarrow \Diamond\varphi$

None of the formulae below is valid in **IMAG**:

$I\Diamond\varphi \rightarrow \Diamond\varphi$	$I\Diamond\varphi \rightarrow \varphi$	$C\varphi \leftrightarrow \varphi$
$I\Diamond\varphi \rightarrow I\varphi$	$IC\varphi \rightarrow C\varphi$	$\Diamond\varphi \leftrightarrow \varphi$
$I\Diamond\varphi \rightarrow \varphi$	$IC\varphi \rightarrow I\varphi$	$\Diamond\varphi \rightarrow C\varphi$
$C\Diamond\varphi \rightarrow \Diamond\varphi$	$IC\varphi \rightarrow \varphi$	$C\varphi \rightarrow I\varphi$
$C\Diamond\varphi \rightarrow C\varphi$	$I\varphi \leftrightarrow \varphi$	$\Diamond\varphi \rightarrow I\varphi$

Considering that **IMAG** has standard metalogical properties, it can be used to settle disputes on the properties of imagination, conception and possibility, as well its interactions. It can be useful to the philosopher lost in the plurality of debates founded in the literature, as for instance those in the books [4] and [8]. Thus, using basic properties of **IMAG**, let's discuss what we consider to be the most interesting properties of it, approaching problems we can find in the literature.

4.1 Descartes-Vasiliev law

Descartes in [2] proposed a distinction between imagination and pure intellection (conception), using the very intuitive example that we can imagine a triangle but we cannot imagine a chiliagon. We can conceive it: understand that it is a figure composed by a thousand sides. Considering this example, it seems very plausible to accept that *imagination implies conception*, if we take imagination as a faculty of generating images while conception as a faculty of understanding a concept (or proposition), even without images. Thus, what we have called Descartes-Vasiliev law is a plausible principle which all logics of imagination should satisfy.

Moreover, Vasiliev's imaginary logic is not a logic of imagination, but this one is the logic of the imaginary worlds. Vasiliev is obviously also concerned with conception.

4.2 Hume's laws

D. Hume collapses the notions of imagination and conception, using both in the same empirical sense. This collapse we cannot accept. However, he is right to state in [6] that both concepts imply possibility. Thus, if we are trying to determine whether a given proposition is logically possible, the best thing to do is to check whether the proposition can imagined or conceived. In this sense, to be able to imagine or conceive φ is a clue to the possibility of φ. It seems impossible to find an intuitive counter-example to these laws.

4.3 Conceivability and imaginability

The reader is now able to distinguish between concepts. Conceivability is an hybrid notion, while conception is a primitive, non-interactive concept. Inside the environment of **IMAG**, it is quite natural to find a counter-example to $\Diamond C\varphi \leftrightarrow C\varphi$. However, if we are in transitive frames, the equivalence holds. The same argument applies to imaginability and imagination. So, the best answer to the question posed in [4] of whether conceivability is a guide to possibility or not, is to state that it depends in what kind of frame our concepts are used. Conceivability and imaginability are good guides to possibility if and only if our frames are transitive. Otherwise, we can find useful counter-models.

5 Conclusion

This text has proposed some new ideas concerning the logics of imagination presented by Niiniluoto. One of the claims of this paper is that Niiniluoto's account is insufficient to deal with imagination and related notions. Other plausible claim of this paper consists in showing that to each object we can associate a proposition and, then, we use this fact to show that any kind of imagination can be reduced to propositional imagination.

The main conclusion of this paper is that imagination and conception are two kinds of possibility. Without interaction axioms relating these notions, they remain the same from the syntactical and semantical viewpoint. This article combined Descartes and Hume's position showing that they are compatible. Using tools from combining logics, we have proposed a combined logic of imagination, conception and possibility showing that the resulting system is sound and complete with respect to combined Kripke frames with special properties in the accessibility relations.

As a very interesting open question to be studied in the future, we can point out how would it be a version of **IMAG** able to deal with contradictions? In this system, we would be able to formalize contradictory conception and contradictory imagination. Thus, we would need to change the underlying logic.

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